The graph of a quadratic function, a parabola, is a symmetrical curve. Its highest or lowest point is called the vertex. The graph is formed using the equation $y = ax^2 + bx + c$. Students have been graphing parabolas by substituting values for x and solving for y. This can be a tedious process, especially if an appropriate range of x-values is not known. One possible shortcut if only a quick sketch of the parabola is needed, is to find the x-intercepts first, then find the vertex and/or the y-intercept. To find the x-intercepts, substitute 0 for y and solve the quadratic equation, $0 = ax^2 + bx + c$. Students will learn multiple methods to solve quadratic equations in this chapter and in Chapter 9. One method to solve quadratic equations uses the Zero Product Property, that is, solving by factoring. This method utilizes two ideas:

(1)   When the product of two or more numbers is zero, then one of the numbers must be zero.

(2)   Some quadratics can be factored into the product of two binomials,

Example 1

Find the x-intercepts of $y = x^2 + 6x + 8$.

The x-intercepts are located on the graph where $y = 0$, so write the quadratic expression equal to zero, then solve for x. $x^2 + 6x + 8 = 0$

Factor the quadratic. $(x + 4)(x + 2) = 0$

Set each factor equal to 0. $(x + 4) = 0$ or $(x + 2) = 0$

Solve each equation for x. $x = -4$ or $x = -2$

The x-intercepts are $(-4, 0)$ and $(-2, 0)$.

You can check your answers by substituting them into the original equation.

$(-4)^2 + 6(-4) + 8 \Rightarrow 16 - 24 + 8 \Rightarrow 0$

$(-2)^2 + 6(-2) + 8 \Rightarrow 4 - 12 + 8 \Rightarrow 0$

Example 2

Solve $2x^2 + 7x - 15 = 0$.

Factor the quadratic.

$$(2x - 3)(x + 5) = 0$$

Set each factor equal to 0.

$$(2x - 3) = 0 \quad \text{or} \quad (x + 5) = 0$$

Solve for each x.

$$2x = 3$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -5$$

Example 3

If the quadratic does not equal 0, rewrite it algebraically so that it does, then use the zero product property.

Solve $2 = 6x^2 - x$.

Set the equation equal to 0.

$$2 = 6x^2 - x$$

$$0 = 6x^2 - x - 2$$

Factor the quadratic.

$$0 = (2x + 1)(3x - 2)$$

Solve each equation for x.

$$(2x + 1) = 0 \quad \text{or} \quad (3x - 2) = 0$$

$$2x = -1 \quad \text{or} \quad 3x = 2$$

$$x = -\frac{1}{2} \qquad\qquad x = \frac{2}{3}$$

Example 4

Solve $9x^2 - 6x + 1 = 0$.

Factor the quadratic.

$$9x^2 - 6x + 1 = 0$$
$$(3x - 1)(3x - 1) = 0$$

Solve each equation for x. Notice the factors
are the same so there will be only one solution.

$$(3x - 1) = 0$$
$$3x = 1$$
$$x = \tfrac{1}{3}$$

Problems

Solve for x.

1. $x^2 - x - 12 = 0$

2. $3x^2 - 7x - 6 = 0$

3. $x^2 + x - 20 = 0$

4. $3x^2 + 11x + 10 = 0$

5. $x^2 + 5x = -4$

6. $6x - 9 = x^2$

7. $6x^2 + 5x - 4 = 0$

8. $x^2 - 6x + 8 = 0$

9. $6x^2 - x - 15 = 0$

10. $4x^2 + 12x + 9 = 0$

11. $x^2 - 12x = 28$

12. $2x^2 + 8x + 6 = 0$

13. $2 + 9x = 5x^2$

14. $2x^2 - 5x = 3$

15. $x^2 = 45 - 4x$

Answers

1. $x = 4$ or -3

2. $x = -\frac{2}{3}$ or 3

3. $x = -5$ or 4

4. $x = -\frac{5}{3}$ or -2

5. $x = -4$ or -1

6. $x = 3$

7. $x = -\frac{4}{3}$ or $\frac{1}{2}$

8. $x = 4$ or 2

9. $x = -\frac{3}{2}$ or $\frac{5}{3}$

10. $x = -\frac{3}{2}$

11. $x = 14$ or -2

12. $x = -1$ or -3

13. $x = -\frac{1}{5}$ or 2

14. $x = -\frac{1}{2}$ or 3

15. $x = 5$ or -9